Flexible investment strategies in distribution networks with DSR: Real Options modelling and tool architecture

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Executive summary

The aim of this report is to provide a comprehensive overview of Real Options (RO) analysis and risk assessment, with focus on potential applications to flexible network investment under uncertainty with Demand Side Response (DSR), and to propose a relevant RO model that could be readily implemented in a spreadsheet tool.

In order to do so, after presenting the main features of options in finance and real options in engineering based on relevant state of the art, we review current approaches that have been undertaken for decision making under uncertainty by National Grid in their Network Development Policy document and by ENWL in their "strawman" RO spreadsheet example. Finally, based on our expertise, experience and studies carried out during this work, and our understanding of ENWL's requirements as to a RO engine to be developed in Excel, we propose a novel RO methodology and describe relevant spreadsheet architecture.

The proposed tool, exemplified in a hierarchical spreadsheet implementation, is based on a multi-layer receding horizon approach to RO analysis of flexible network investment under uncertainty with specific inclusion of DSR. The model is organised in terms of strategy (layer 1), long-term scenarios (layer 2) and short-term Monte Carlo simulations (layer 3), thus bringing together and deploying the optimal features of different RO approaches as fit for the purpose of this work. The proposed tool can be flexibly adapted to take decisions on a regular basis (for instance, every year), and the underlying model features the upsides of the receding horizon approach successively deployed in the engineering applications of optimal control theory and also makes up at the same time for some limitations that implementation in a relatively simple tool brings.

Different metrics and decision criteria are discussed and can be implemented in the tool, based on probabilistic representation of relevant random variables and allowing specific consideration for financial and physical risk analysis for different strategies to be considered.

Useful outputs of the proposed tool may include:

- Optimal investment strategy for the current year (decision time), to be reassessed with receding horizon every year in the light of the projected scenarios and estimated uncertainty.
- Ranking of the considered decision strategies based on the input intervention alternatives (the "design options") by different metrics (expected cost, expected cost weighted with risk metrics, least worst regret, weighted least regret, and so on).
- Detailed breakdown of the probabilistic distribution of costs of each strategy in each scenario plus the overall probability weighted distribution of costs for each strategy, so that fully informed and transparent decisions can made.

The tool can be applied in various ways besides determining optimal investment strategies, amongst others for optimal DSR pricing and to quantify financial and technical risks associated to specific interventions and suitability of an asset portfolio to meet relevant techno-economic requirements set out by the Regulator.

1. Introduction

The primary aim of the work summarised in this report is to provide strategic indications to ENWL as to the development of a "Real Options" (RO) method, to be later implemented in a relatively simple spreadsheet tool, which is capable to give a techno-economic and financial assessment of different options for capacity increase against load scenarios. In particular, specific focus will be put on describing competition and complementarity between network asset reinforcement and Demand Side Response (DSR) alternatives. The latter case can for instance be represented by the C2C methodology for post-contingency DSR, but in general any type of DSR interventions could be considered in this context.

The critical points to justify the need for this work is the increasing uncertainty in forecasts of net electricity demand changes, particularly due to the arrival of low carbon distributed technologies both on the demand side (e.g., electric heat pumps and electric vehicles) and the supply side (e.g., photovoltaics, wind and cogeneration). This uncertainty makes it difficult to make long-term investment decisions that might potentially lock into future stranded assets. Hence, having available *flexible* investment options that might easily be converted into something else in the case the future were to develop differently from expected would be extremely valuable. In this respect, while flexibility is commonly exercised in real-world management decisions, the investment analysis tools that are traditionally adopted do not properly reflect such conditions. In particular, the Discounted Cash Flow (DCF) methods commonly utilised for Cost Benefit Analysis (CBA) as in Ofgem's CBA spreadsheets for RIIO-ED1 implicitly assume that:

- Investments are reversible, while network investment projects suffer from large sunk capital costs.
- Cash flows are deterministic and certain for the entire lifespan of the project, while future forecasts/scenarios are highly uncertain.
- If investment is not made now, it is foregone forever, while this decision can be postponed or changed based on "active" management.
- A single discount rate is applied to all cash flows, while capital investment (typically managed by the firm) may be much safer than future cash flows based on alternative options.

In particular, while DCF analysis assumes that the investment decision is a now-or-never decision and if the NPV is negative then it is foregone forever, in reality the investment can be delayed until part of all of the underlying uncertainty is resolved. This can avoid getting locked in with stranded assets in unfavourable scenarios. For example, if during the planning stage demand does not grow as expected, the decision maker might not invest right away but rather wait-and-see the out-turn of demand, and decide either to invest if demand goes above a certain threshold or to keep going with DSR and possibly wait more to see the development of uncertainty. Extensive examples on alternative developments of potential futures turning out into completely different options are reported in the first C2C project

Deliverable on Work Package 2 [Cesena and Mancarella, 2013]. Hence, there is a clear flexibility value in having the opportunity to delay the investment. In the same way, there is potential value in accelerating, abandoning or altering investment decisions based on how the future develops. Ignoring this flexibility causes many investment decisions, and particularly the highly flexible ones, to be undervalued and in this case never carried out, at the cost of potential asset stranding.

While the above theoretical and practical gaps between traditional DCF-based investment analysis methods and flexible decision making perspectives cannot result in correct valuations, it is to some extent possible to deploy tools for decision making under uncertainty which have been developed in other disciplines such as finance, and model the main uncertainties so as to reflect as closely as possible how a human decision maker carries out the best decision at a given time, once current conditions and possible future ones are known. Real Options theory, based on the rationale of financial option pricing methods, was developed precisely to fill these gaps, so that flexible investment strategies can be developed to resemble realistic decision making processes. In particular, the strategy which is ultimately selected should be flexible enough to remain acceptable over a range of possible future scenarios, taking full account of the future adaptations that will be available as new information arrives over the timeframe of the analysis. In addition, the tool that is aimed at should be flexible enough to be reusable while uncertainty is resolved and the future unfolds. In this context, the advantage of selecting a Real Options approach to deal with uncertainty is that in RO analysis the task before us is to choose a 'best' strategy among the list of possible flexible strategies, taking into account the range of possible futures and our subsequent adaptations to them, together with any information we may have about the relative likelihoods of these futures. In particular considerations such as the average future cost (or equivalently benefit) and the potential variability in this future cost are important in selecting the best strategy. On the other hand, while considering the likelihood of possible futures to occur and weighting them in a relevant model provide important insights relative to considering only one future, such an approach, based on averaging, does not provide information about *retrospective* views of the risk of undertaking particular decisions. For instance, taking into account the obvious fact that we will only experience a single future in reality, the amount of retrospective 'regret' we may potentially face in each *particular* future, irrespective of how likely or unlikely that future may seem to us at the present time, could also potentially be an important factor for inclusion in our analysis and in the RO approach as well, and will be expanded upon throughout this report.

This report is organized as follows. In Section 2, we discuss general aspects of decision making under uncertainty. Starting from the concept of financial options, the focus is on real options modeling with engineering applications, and we provide a comprehensive critical review of possible RO evaluation approaches and their suitability for network investment problems. Risk analysis and relevant techniques are also discussed. We review in Section 3 the National Grid's Network Development Policy document (including discussion on the "Spackman" approach to network investment costing) and the ENWL's "strawman" example spreadsheet, and we give relevant comments and feedback. Section 4 is the "core" of this report, where we propose a novel multi-layer RO model for flexible network investment valuation and we give indications as to what a possible architecture for implementation of the model in an Excel workbook could be. Section 5 contains the concluding remarks.

2. Decision making under uncertainty and real options valuation: state of the art and understanding based on previous work

The objective of this Section is to introduce the general understanding on techniques for decision making under uncertainty and RO according to the work carried out so far at the university, in line and additionally to what was presented at the November workshop.

2.1. Options in finance

Financial markets are well known to be characterized by uncertainty. Different securities such as stocks, bonds, commodities and so forth have different levels of expected or average performance. At one extreme, US government treasury bills were long regarded as being risk-free investments. Stocks in companies such as Apple, in contrast, offer uncertain returns and therefore must offer the investor a higher expected return than US government treasury bills (at least over the long term), as otherwise very few investors would regard such a stock as attractive in comparison with treasury bills. Since the typical investor's choice is between a huge number of different market securities each with different expected returns and volatility of returns, the study of optimal trading in financial markets could therefore be reasonably regarded as the study of financial market uncertainty.

The study of uncertainty is approached by looking for structure, that is to say, repeating patterns or tendencies that are expected to persist in future. This should not be confused with seeking to predict the future exactly, rather it is concerned with understanding the ranges of possible futures and their relative likelihood. In financial markets it was already observed by Bachelier in 1900 [Bachelier, 1900] that the structure of stock market returns was similar to the structure of certain abstract mathematical objects. This observation opened the door to the study of mathematical finance, since it now argued that studying stock market uncertainty was equivalent to studying a branch of mathematics (now called stochastic process theory; independently, Einstein himself studied stochastic processes in the completely different context of physical diffusion).

The simplest option contract is perhaps the European option, which gives the holder the right but not the obligation to trade a specific stock, at a specific price, at a specific future point in time. The rational option holder therefore only exercises her option if it is economically favourable to do so on this so-called expiry date, otherwise the option expires worthless. In this way, the study of option values is not simply concerned with understanding the repeating patterns in stock price behaviour, but additionally it is concerned with understanding the value of the flexibility offered by the option contract. Although Bachelier provided option pricing formulae, it was not until the rediscovery of Bachelier's thesis by Samuelson in 1965 and subsequent work of Black, Scholes and Merton [Black, 1976] that it began to be more widely understood how to put a value on flexibility in financial markets. By making rather strong, but nevertheless clearly acknowledged, assumptions about the structure of financial markets Black, Scholes and Merton were able to obtain straightforward expressions for the value of simple financial options. Their formulae had not only the attraction of being simple to write down and use, but also of requiring no further expert input: they can be regarded as algorithms, which simply observe today's stock market prices as input and return the option values as output, with only a single, relatively easily estimated parameter (the volatility) for the user to provide. It is therefore with their contribution that the theory of mathematical finance and option valuation began to gain traction.

The success of mathematical finance must be balanced by the price of this success, which is a possible temptation to trust its formulae without acknowledging the strong assumptions which make it valid. Of course these strong assumptions are so well-known among finance experts that while they were carefully stated in papers of the 1970s, much less care was typically exercised tens of years later when, for example, the nascent field of Real Options

Analysis began the task of translating the insights of mathematical finance from the financial markets into the field of real commercial projects. It is beyond the scope of this report to give a full review of either the typical assumptions made in the field of mathematical finance, or of the size or severity of the potential consequences of neglecting to check the validity of these assumptions in the particular setting of Real Options Analysis for distribution networks; instead we will highlight these issues on an as-needed basis. From the opposite side, the qualitative insights provided by making assumptions that are known not to hold (but acknowledged as such) may potentially provide useful guidance and rules of thumb that would not be clear from a more complex and detailed analysis. However, we note that when large investment decisions are taken using Real Options Analysis then the decision maker should take proper care to satisfy themselves that the assumptions of the analysis, including the application of any analysis from mathematical finance, are understood and clearly acknowledged.

2.2. Real Options in engineering

2.2.1. When to apply Real Options analysis?

Just as financial options provide a contractual right to take an “optional” action which will be exercised by the rational holder only if favourable conditions occur, engineering projects (and network projects in particular) are somehow naturally “packed” with (investment) decision options, such as for instance the use of demand side response (DSR) as an alternative to network reinforcement, securing additional space in a substation to place an additional transformer if needed, opting for like for like replacement or with a larger asset to avoid subsequent early load-related reinforcement, staging the investment over time through two smaller ones, and so on. Hence, in general there are a number of questions that can be associated to engineering investment problems, which can be categorised schematically as “what” (and “how much”) and “when”. However, for a particular flexibility to bear a RO value, there are a few conditions that need to be satisfied, namely:

- Whole or partial irreversibility of investment costs. In fact, if all investments were perfectly reversible there would be no need for RO analysis since all strategies would be seen to be equivalent by simply reversing any decision that would turn out suboptimal and investing into the optimal ones instead. In the context of network investment this condition essentially always applies, since even when an investment decision can be reverted or the investment can be cashed into other forms, there is typically an irreversible loss of capital due, for example, to high sunk costs or depreciation of the used asset’s value compared to its investment cost. The irreversible part of the network investment is therefore that fraction of the investment cost that cannot subsequently be recovered (for instance, by redeployment at alternative locations), together with any consequentially lost costs and benefits.

- Uncertainty in future movements of key variables, which significantly influence future cash flows (perhaps by affecting the adequacy of current investments). In the context of network investment these may include uncertainty in demand growth, uncertainty in DSR to get contracted or to be available when needed, changes in the way cash flows are calculated for regulatory purposes, and so on. Alternatively, if the future is known, then as indicated above DCF analysis is adequate to make decisions. In this respect, it is worth noticing here that RO analysis always builds upon some type of DCF model that takes into account time value of money, for instance one DCF model for each possible scenario that might be realised. Hence, if a DCF model cannot be built for some reason, neither can a RO model. Likewise, multiple possible futures need to be considered in the RO analysis, since otherwise the future could be predicted perfectly and therefore DCF would work perfectly.

- Strategic flexibility to change the course of action, including changing the project timing: in particular, the decision maker can usually take intermediate actions such as to invest, abandon, defer, expand, contract, in response to uncertainty (for example, from unexpected events such new regulation, energy policy incentives, etc), and is free to take these actions and make the relevant adjustments to investment at a number of possible times. If an investment needs to be made now or never, there is no flexibility and no room for real options analysis: again DCF works well under this condition.

As a result, it is possible to summarise the above statements by saying that the correct value to quantify a flexible strategy in the presence of uncertainty and irreversibility is

Project Value = *Traditional Net Present Value* + *Real Option Value*

As will also be discussed in the proposed RO model in Section 4, this RO value is calculated as an average over the possible futures, with a certain strategy that could be used or not used in a given scenario, but whose value would always be positive or at worst zero (if the option is not "exercised"), but never negative. Hence, the option value, calculated as an average of nonnegative values over the different scenarios, will never be negative either.

2.2.2. Real Options and flexibility

Real options do not "create" flexibility, but highlight in a quantitative way the *value* of the flexibility that is available in decision making, which is particularly important in a network investment context. As such, it is not that RO thinking favours more flexible projects, but simply highlights the benefits from flexibility that other techniques such as those based on DCF cannot. The result is that RO analysis allows a fair comparison between flexible and inflexible network investment strategies by giving both their fair value. In particular, the flexibility that is highlighted is in response to uncertainty: future conditions can turn out worse than anticipated but also better than anticipated, and RO provides a methodological approach to account for the fact that in reality, decision makers will seek to take advantage of future better conditions when they occur and, conversely, will seek to minimise the impact of future poorer conditions should they arise.

Two fundamental types of flexibility exist in engineering projects, which together lead to the concept of "strategy" in the context of this work (see Section 4):

- Flexibility in the *timing* of the decision. The typical example from finance is the American Option, which provides the holder with the right but not the obligation to trade in a specific stock, at a specific price, at *any* time until a given and fixed expiry date. In this way, the American option may be regarded as a European option with added timing flexibility. An example in the context of network investment is the ability to use DSR while waiting to see if demand rises, rather than making the investment intervention now. In addition, projects may have multiple inter-temporal options, that is multi-staged decisions can be made over time ("when" and "what"; the simplest example from finance would be holding *two* American options which can be exercised independently of each other).
- Flexibility in the *design* of the project: just as there are a variety of financial options available with respect to *timing*, there are also a variety of financial option designs (Vanilla, Binary, Barrier, Bermudan, Asian, and so on). However, such 'exotic' financial option designs have found rather few parallels in engineering studies. It is therefore preferable to distinguish real options design from financial option design, and in engineering problems: (i) to simply list all possible *designs* available to the decision maker in the particular problem under analysis; (ii) for each design, to list each possible *strategy* (which generally speaking is a *combination of design and timing*) that could be used

under that design; and finally (iii) to calculate the RO value for each individual strategy. For example, given the same long term “final” *design* for network replacement represented by the combination of DSR and asset reinforcement, different *strategies* might differ in terms of when each intervention would be carried out and through which “trigger points” (for instance, DSR first until 95% capacity is reached and then reinforce the asset, or DSR for a certain number of years and then reinforcement, and so forth).

2.3. Modelling uncertainty and flexibility in Real Options

2.3.1. Uncertainty and time flexibility

Since flexibility is fundamentally the capability to respond to change, thus enabling optimal active management, the first critical step in RO analysis is to model the significant uncertain variables in the problem. In traditional option theory which mainly deals with stock market uncertainty, prices are modelled through various stochastic processes that have been established as successful representations of stock movements over time.

In engineering applications, standard Brownian motion (or Wiener process) or Geometric Brownian Motion (GBM), as well as mean-reverting processes (particularly to model electricity and gas prices) have been used in the literature. However, for RO applications a few substantial issues arise in utilising such stochastic processes. The first and most important is that while there are very good arguments to use for instance GBM processes for stock prices, there is no guarantee that the model of an engineering variable such as for instance peak demand in a network should follow such a process. In addition, mathematically it becomes very complex to model such stochastic processes when there are more than two correlated uncertain variables, thus potentially negating the mathematical benefits of such an approach. Last but not least, most of these processes are difficult to justify for the long-term time-scale (i.e., investment decisions) since they implicitly make strong assumptions about the future such as constant mean and volatility (which may typically not be the case over for instance 30 years). However, suitable modifications can be made to and have been proposed for these basic models to make them more suitable to specific applications. Also, classical stochastic processes can be suitable to generate short-term variation attributable to uncertainty or to generate “noise” around longer term projections which could for instance be based on scenarios (see below). Such approaches to short-term uncertainty generation and the modelling of errors will be used in the model proposed in Section 4.

Another typical approach that can be proposed to model uncertainty is based on scenarios, whereby instead of continuous random variation over time the uncertain variable can follow specific time trajectories, which are often associated to probability of occurrences. The scenario approach takes to the extreme the discretization of the continuous process formulation that is carried out in option theory through for instance lattice models. In fact, the original model developed by Black and Scholes for option pricing could be applied only to European-type options that could be exercised only at maturity. In order to model American-type options with possible early exercise other approaches were developed, such as continuous models based on more complex theory (the theory of ‘optimal stopping’) which do not provide simple valuation formulae, or alternatively simpler discrete models that provide a practical approach to modelling intermediate time stage decisions. Since there may be high value in delaying investment and waiting for some uncertainty to be resolved, RO models applied to engineering and in particular the long-term nature of investment analysis means that discrete models that can model long term uncertainty are better suited, especially if long term trend changes can be more clearly incorporated (as in scenario-based models). When the problem needs to be modelled as a staged process, particularly if several options exist over an investment lifespan, the lifetime is divided into discrete number of periods at which milestone decisions can be made.

2.3.2. Design flexibility

With respect to modelling project design flexibility, which as mentioned above has few useful parallels in conventional option theory, here the value indeed lies in designing a system in a suitably flexible way. Further, as described above, the issues of *design* and *strategy* are closely interwoven. For instance, one can think of the high value in optimally designing a project beforehand by optimising the capacity of a line or transformer to reinforce. In this way, by increasing future flexibility, design can also enable a greater set of possible strategies. Consider, for example, expansion of one large-capacity line (with no future flexibility) as opposed to adding one smaller-capacity line (with the flexibility to expand through another one later if needed). In this context, RO analysis essentially takes proper account of such possible future economies of scale, together with a proper adjustment which recognises that these economies of scale will only be utilised in certain favourable scenarios. On the other hand design can also introduce dependency between today's options and future ones, and hence reduce the number of available strategies; for instance, when adding one large line makes obsolete the option of adding another small line. Clearly, though, increasing the number of design *possibilities* increases the number of available strategies. While this can only increase the RO value calculated by the model, there is a resultant increase in the complexity of the RO model which must also be considered and in extreme cases an optimisation model could be required on top or the RO model, which is either to be embedded within the RO analysis engine or whose results need to be provided endogenously to the RO engine.

2.4. Solving RO problems

A short overview of solution techniques for RO problems which have been proposed in the literature will be given in this Section. While it is out of scope to discuss details, focus will be put on the suitability of different approaches for engineering problems and particularly to network investment problems as the ones under study, also considering implementation complexity issues.

Starting from the models that have been traditionally used for valuing financial options and which have been extended to RO applications in a more or less straightforward fashion, it is possible to mention:

- The Black-Scholes equation;
- Finite Difference methods;
- Lattice Methods;
- Monte-Carlo simulations.

2.4.1. The Black-Scholes model

The Black-Scholes formula [Black and Scholes, 1973] is probably the most famous and recognised model in option theory. The main pros of its application (and also the reason for its success) are the fact that it provides an analytical solution and at the same time it is extremely simple to implement, only requiring to input relevant numbers into the equation, so to speak. However, several major cons apply, namely: the fact that the decision is made only at the time the option expires (that is, the option is of the European type, and cannot thus model optimal invest timing); only one uncertainty, or at most two correlated uncertainties can be modelled; uncertainty must follow a GBM process with constant mean and volatility; and interdependencies between different options cannot be considered. On top of these "structural" downsides, one should also mention that due to its simplicity this formula has often been abused by its application far beyond the purpose for which it was developed.

2.4.2. Finite difference methods

Finite difference methods have been applied to solve RO problems that can be formulated in an analytical way as partial differential equations starting from suitable continuous stochastic processes. There are several pros in such applications, in primis the fact that Finite difference methods can value very complex options in an accurate way and in continuous time. However, for practical RO engineering applications it is not always easy to define an underlying equation to solve and finite difference methods suffer from the 'curse of dimensionality' with respect to the time needed for computation so that, like the mathematical Black-Scholes model, they are limited to one or two uncertainties and may be challenged by interdependencies between different options.

2.4.3. Lattice methods

Traditional lattice methods

As already mentioned above, traditional lattice methods [Mun, 2006] were developed with the idea to discretise the continuous solution provided by the Black-Scholes formula so that intermediate time stages could be accessed and therefore American-type options could be valued too. Lattice methods can be fast and simple to implement, particularly for spreadsheet applications which may take advantage of dynamic optimization techniques. Also, they allow consideration of multiple time periods and investment timing decisions (for example when the option is “American”). However, as for the Black-Scholes model, lattice methods rely implicitly on the specification of parameters such as volatility and mean (which can be an issue for long term investment that may as a result have to be based on specific scenario considerations), they become quickly intractable in a spreadsheet implementation when the number of uncertainties and time-steps grows.

Extended Lattice Methods: Wang Lattice Methodology

Extension of lattice methods to non-recombining trees to address engineering applications was recently proposed in several works by Wang [Wang and De Neufville, 2004], whose formulation thus allows considering interdependencies between options (as the decision tree is non-recombining) and also optionality in the project design in order to create greater flexibility and then value with respect to timing only. However, again Wang’s model considers GBM stochastic processes only (constant volatility and mean), and it becomes intractable with increasing number of uncertainties and time-steps (although this is a limitation of all lattice based models). In addition, the additional value provided by design optionality comes at the cost of complexity, as optimisation techniques need to be put in place on top of the RO model, so that the problem is generally cast as a mixed integer non-linear programming problem whose solution may be not straightforward.

Extended Lattice Methods: Cesena Lattice Methodology

The model by Wang was improved further by Cesena [Cesena, 2012], with the possibility of using any type of process to model uncertainty (including discrete and scenario based ones), embedding design options at different levels of the decision tree (while Wang only considers them at the pre-screening stage), and adopting any optimisation for determining optimal time of investment (based on extended search-like algorithms). However, again this comes at the cost of complexity, with the model requiring design optimisation stages and becoming potentially intractable the greater the number of uncertainties and time-steps.

Extended Lattice Methods: Extensive Search Methodology

Extensive search algorithms may be seen as simpler cases of Wang’s and Cesena’s models, where there is no optimisation applied at different stages. They can consider interdependencies between options (the decision tree is non-recombining) and use any type of process, although in most cases scenarios will be given deterministically and attention needs to be paid to the decision nodes that are considered. Given its simplicity, such methods could be used to consider relatively simple network investment assessment problems.

2.4.4. Monte Carlo methods

Monte Carlo simulations: Traditional Methodology

Since they are based on simulations, Monte Carlo methods are powerful techniques that can be used to handle RO problems with multiple uncertainties and with different probability distributions and stochastic processes of any type. On the other hand, some "off-the-shelf" Monte Carlo methods cannot consider multiple time periods, can only use a single discount rate (differently from extended lattice methods, which in this light provide more flexibility to model risk, as discussed later), and cannot consider interdependencies between different options. However, Monte Carlo approaches are so flexible that there are various way to adopt them in a RO context for specific purposes such as to test probabilistically the implications of a given strategy under given scenarios. An application in this direction will be included within the RO approach proposed in Section 4.

Monte Carlo simulation: Datar-Mathews methodology

A recent methodology proposed by Datar and Mathews [Datar and Mathews, 2007] extends the classical Monte Carlo methods by adopting two discount rates, namely, one for risky cash flows and another for safer investment. In this way, by modelling risk in a more appropriate way, the model takes further advantage of upside benefits, while minimising losses on the downside. However, again in its standard implementation this approach cannot consider multiple time periods and interdependencies between different options.

Monte Carlo simulation: Least-squares regression methodology

The limitations of Monte Carlo methods of not being able to consider multiple time periods and therefore model American-type RO have been overcome by the so-called least-squares regression methodology, whereby regression algorithms are applied within Monte Carlo simulations to access intermediate time decision stages as in lattice methods. However, a main drawback of standard Monte Carlo methods not being able to consider interdependencies between different options still remains.

2.4.5. Final consideration on the different RO approaches

As should be clear by now, engineering RO approaches focus on decision flexibility to make potential investment only if it is worthwhile, otherwise, in financial jargon, the option is never "exercised”. In this light, all the models discussed above are capable to value options that can be exercised at expiry (that is, with a given investment time) while not all models are capable to value options with multiple investment stages or to identify the optimal investment time. Based on the above discussions, specific problems that can be addressed by different techniques (in their standard implementations) are summarised in Table 1.

Table 1. Major features of different RO techniques

	BS	FD	L	W	C	ES	DM	MC	LSM
Multi-stage, with strategic decision roadmap			x	x	x	x		x	x
Assess the best time to invest			x	x	x	x		x	x
Consider interdependencies between options				x	x	x			
Can model more than two uncertainties						x	x	x	x
Have the flexibility to explicitly use scenarios					x	x	x		
Do not require design optimisation	x	x	x			x	x	x	x

BS: Black-Scholes; FD: Finite Difference; L: Lattice; W: Wang; C: Cesena; ES: Extensive Search; DM: Datar-Mathews, MC Monte Carlo; LSM: Least-square Monte Carlo

2.5. Risk analysis and robust optimization

2.5.1. Need for risk analysis

While RO analysis is fundamentally associated to weighted averages, particularly when coupled to scenarios modelling long term uncertainty, a multi-criteria approach can be incorporated into the RO-based decision making framework to take into account risk, which in general terms can be defined as the consequences of unexpected events taking into account their probability of occurrence. In particular, several criteria (including for instance the risk of occurrence of certain "bad" outcomes, cost associated to occurrence of certain "bad" outcomes, regret of the selected strategy with respect to the scenario that actually materializes, and so forth) may actually be evaluated separately, and then these results can be considered side by side in decision making.

To justify the need for risk analysis, let us consider for instance the everyday example of carrying an umbrella: if I carry an umbrella and it does not rain then I regret the minor inconvenience of needlessly carrying the umbrella, whereas if I do not carry an umbrella and it does rain I regret the major inconvenience of being drenched. The *least-regret* strategy for the umbrella is therefore to always carry an umbrella; however, in reality people tend to carry an umbrella only when there is a significant likelihood of rain. A possible interpretation of this behaviour is that the probability-weighted decision (i.e., no umbrella) is used when the chance of rain is very low, whereas the least-regret decision is employed when the chance of rain is significant (even if the chance of rain is much less than 50%). Under this interpretation, both criteria (average cost and least-regret) are calculated separately and then considered side by side when deciding on the umbrella.

2.5.2. Various approaches to risk modelling

While it is outside the scope of this work to provide a comprehensive overview of risk modelling techniques, it is possible to give some general considerations for applications to RO analysis and particularly to the approach and tool that will be proposed in Section 4.

Deterministic analysis, risk premiums and discount rates

A classical way to take into account risk in a deterministic environment and in a traditional DCF context is to incorporate it into the discount rate factor. In fact, DCF models operate by discounting cash flows to the present considering *both* the *time value of money* and the *uncertainty in the expected future cash flows* through a single *risk-adjusted discount rate r*, namely composed of a risk-free interest rate (possibly adjusted for inflation) r_f (corresponding for instance to the rate of return on Government securities) to take into account the time value of money, and a risk premium *RP* to take into account uncertainty and risk: $r = r_f + RP$. Hence, both investment and variable cash flows are typically discounted by this same discount rate *r*, often associated to the WACC. However, in reality the risk involved in any given strategy may be multi-faceted and detailed modelling should account for this. As an example, consider the risks involved in the following strategies:

(i) Make a capital investment immediately. The price risk in this case may be negligible, for example if the asset is purchased today at a known price.

(ii) Make a capital investment in 10 years. There is now price risk due to inflation if the contract is to be signed at a future date rather than being signed today. The total risk in this case (ii) is therefore greater than that in case (i).

(iii) Employ a relatively new DSR technology now in order to defer capital investment for as long as possible and then perform capital investment when DSR becomes exhausted. Here, inflation risk applies to future DSR payments similarly to case (ii) above; however, as the DSR technology in this case is relatively new, there may also be physical risk that average DSR availability may be less than 100% - that is, the average availability level that will actually be realised by this new technology is not known at the present time, for instance based on the fact that not enough DSR capacity might be available when needed or could not be contracted on time (it might additionally be the case that the network is stressed without having DSR at full load, for instance due to uncertainty in peak load forecast). This average DSR availability level may for example increase over time as experience is gained from both a contractual and a control point of view. For ease of analysis, and depending on the particular constraints of the DSR solution, this physical risk could potentially be modelled in financial terms as a change in the cost of 1MW of *available* (rather than contracted) DSR. Whether the physical risk is quantified in physical or economic terms, though, it is clear that the total risk in this case (iii) is greater than in case (ii). Further discussion is provided in Section 4.

This example makes clear that if we choose to use the quantity RP as a simple means to account for risk, then the value of RP should vary depending on the nature of the particular strategy under consideration; further, the *size* of this variation in RP should depend on our beliefs about the range of possible futures. Equivalently, the use of a single fixed value of RP (equivalently, a single discount rate) across all strategies in the analysis regardless of their nature, while having the attraction of simplicity and convenience, can lead to oversimplification and hence misrepresentation of Real Options values. It is important to note that strategy (iii), which may have the highest risk premium, could nevertheless still be the *best choice* of strategy as it is the only strategy which allows the possibility of completely avoiding the capital investment in the case that future demand turns out to be low.

Among the possibilities to use discounting for risk analysis, as mentioned above there is the option to discount differently fixed (for instance on the basis of the WACC) and variable costs and benefits (in the case they are uncertain because of DSR). In the case of risky future benefits, there might even be the possibility of using a very low and even *negative* discount rate at the extreme (that might change with time to take into account changes in risk, for instance in the case of additional DSR required and potentially increasing problems to incrementally contract customers). Certainly the choice of the approach to undertake and the most appropriate value of discount rate are subject to discussion and needs to be based on experts' assessment. Sensitivity studies (not to be limited to the variable costs discount rate, but for instance also to fixed costs when in the future), for instance through a "tornado diagram" (whereby the variations of a given model output, e.g., overall expected cost of a project, with respect to certain changes in different input factors are plotted along a vertical axis so as to resemble a *tornado* shape), can provide important insights into the analysis and particularly as to what parameters the outcomes are most sensitive to.

Probabilistic analysis and "tail" indicators of risk

When dealing with uncertainty in a probabilistic fashion (for instance, based on Monte Carlo simulations in the presence of a stochastic process, or on the basis of scenario analysis), different risk measures can be adopted, which again are mostly coming from finance disciplines. Amongst others, besides the classical *Variance*, which gives an indication on the dispersion of a random variable with respect to its mean, the *Value at Risk* (VaR) and the *Conditional Value at Risk* (CvaR) have been recently proposed. One attraction of VaR and CvaR is that they are one-sided, in that they give information about the likelihood or severity of unusually high costs, while unusually low costs are not regarded as contributing to financial risk. In contrast, less sophisticated measures of variability such as variance are two-sided so that both unusually low and unusually high costs contribute equally to variance.

VaR is one of the most well-known and widely used methodologies to assess risk exposure. It is intended to mark a boundary between "normal" costs and "extreme" costs and as such it specifies a cost threshold. For a user-defined exceeding probability (or confidence) level *α%* (for example 5% or 1%), losses greater than the VaR threshold are judged to be sufficiently unlikely that they occur only with probability *α%*.

Notwithstanding its popularity, the VaR has been questioned as an appropriate measure of risk for two main reasons. Amongst other reasons, there is the fact that since the VaR is merely a threshold to separate normal costs from extreme costs, it is not designed to indicate the average size of an extreme cost when one occurs. Therefore the *CvaR*, which may be interpreted as the expected size of an extreme cost (measured by the VaR threshold), was proposed as an alternative (or companion) risk measure to VaR.

Decision theory models and robust optimization

An alternative framework to deal with uncertainty and risk preferences particularly in the presence of scenarios is *decision theory*, which is used to enable the decision-maker to maintain a significant level of interaction between interventions strategies and risk control for different scenarios to be analyzed. In decision theory, different objective functions (to be minimized, in the case of costs) and different decision criteria (for instance, minimum expected value, minimax weighted regret, mixed optimist-pessimist criterion, and so on) can be considered. The results from adopting specific intervention strategies are then expressed considering the different scenarios used to model long-term uncertainty and their associated probability weights, and the decision maker adopts the relevant decision criterion to select the final strategy. Amongst the most used decision criteria, it is possible to mention (in the case of cost) the "*minimax weighted regret*" criterion, where the optimal strategy selected across a number of scenarios is the one that minimizes the maximum regret felt by the decision maker, after verifying that, given the outcomes obtained (hence, *ex post*), the decisions made *ex ante* were not the optimal ones. If no weights are applied to the relevant scenarios and only the "absolute" maximum regret is considered, this approach basically turns into a *robust optimization* approach in the presence of scenarios, whereby the decision maker would hedge themselves against the worst possible outcome of the futures, and the selected strategy would be a conservative one. This approach coincides with the "least worst regret" approach that National Grid has recently adopted, as discussed in Section 3.

In addition, when for each scenario a probability distribution of the relevant indicator, say the Net Present Cost, is available (for instance because Monte Carlo simulations have been considered to model small-scale uncertainty, as done in our proposal described in Section 4), the criterion may for instance be based on the “expected value” (as the probabilistic metric) of the distribution in each scenario, but it may also generally be referred to any α% exceeding probabilities of the random variable (similarly to the definition given above for the VaR). A comprehensive framework to deal with large-scale uncertainties or over the long-term (through decision criteria approaches) and small-scale variability or over the short-term (through Monte Carlo simulations) is reported in (Carpaneto et al, 2011a and 2011b).

Further considerations on risk modelling and indicators will be discussed in Section 4 with respect to the proposed RO modelling framework for DSR valuation.

3. Considerations on existing work by National Grid and ENWL

The purpose of this Section is to provide feedback on work recently carried out in the context of decision making under uncertainty and in particular on the Network Development Policy (NDP) document [NGET, 2013] by National Grid Electricity Transmission (NGET) and the RO spreadsheet "strawman" example developed by ENWL. The final aim is to leverage on work already done to identify upsides and downsides of the approaches undertaken so far and therefore to provide useful insights into the proposed RO methodology.

3.1. The Network Development Policy (NDP) from National Grid Electricity Transmission (NGET)

3.1.1. The NGET's Network Development Policy document

National Grid Electricity Transmission (NGET) has recently proposed a scenario-based approach in its latest Network Development Policy (NDP) document for transmission system investment strategy. The approach has been approved by Ofgem as sound in response to the investment challenges that a transmission operator faces. More specifically, NGET needs to balance the risk of transmission reinforcement early investment (which might result in sunk costs and stranded assets if certain generation and demand expectations do not realise) against the risk of a late investment, which includes congestion costs. This needs to be done considering the possible uncertainty in future demand and generation, and therefore National Grid has introduced a number of scenarios that represent a manifold view of possible futures as opposed to a single "best" view. In particular, three scenarios (which are not forecasts), namely, Slow Progression, Gone Green, and Accelerated Growth, were considered in 2011 and 2012, subject to continuous change based on consultations with relevant stakeholders. More recently, sensitivities to two key scenarios (Gone Green and Slow Progression) which turn out into new scenarios themselves were considered.

Each scenario is analysed against a number of interventions (always including the "do-nothing" strategy) that are supported by inputs from external network analysis models and that are assessed based on the NPV of the costs of investment, constraints and losses. More specifically, network investment requirements are identified based on the most binding between thermal, voltage and stability constraints and considering the incumbent security criteria. All investment solutions are always of a range wide enough to include both small scale and short lead time and large scale and long lead time solutions.

The investment options are considered over 45 years (this is generally Ofgem's suggestion for network investment studies) of projected data, and include various different transmission "strategies", that is, combinations and timings of transmission solutions, until the lowest cost one is found for each and all scenarios. Considering the financial substance of investment needed and the volume of the uncertainty involved, risk analysis is critical, which manifests itself in the decision on optimal timing for investment. In fact, by waiting, new information that could be revealed might confirm or not the need for a certain investment, thus increasing its expected value against alternatives and reducing (or eliminating at all) the associated risk of stranded assets. On the other hand, considering the long lead times for investment, waiting too long could mean increasing congestion cost risk in some scenarios. In addition, the optimal strategy can be different for each scenario, that is, the possible candidate solutions are competing across scenarios and there thus is a "risk of regret" if a strategy is chosen but a different scenario effectively materializes.

In the light of the above, the conventional CBA approach used by National Grid so far is replaced by a new framework that accounts for optimal timing and risk adjustment based on decision theory and more specifically on a "least-worst regret" criterion. This rule calculates for each scenario the "regret" of adopting a certain strategy with respect to the optimal strategy in that scenario, and then selects the optimal strategy as the one minimizing ("least") the maximum ("worst") regret across all strategies and scenarios (see Section 4.1.5 for an illustrative example in the context of our model proposal). In this way, a representation of risks and benefits of all possible strategies under the various scenarios is pictured and a robust risk-inclusive decision is made from the point of view of minimizing possible regrets. As discussed above, this approach can also be seen as a robust optimization approach, where the decision maker, who is intrinsically risk-averse, follows a risk-minimising strategy to decide between competing investment options and the staging of investment commitments rather than, for instance, one that would minimise the expected cost across the multiple possible scenarios. To confirm the robust optimization nature of NGET's approach, extreme scenarios with high local generation and insufficient network capacity and low generation and stranded network capacity are considered too. One of the limitations of the approach, as mentioned in the document, is that the new information is assumed to be available at a certain point in the future, against which the options are assessed in terms of least regret. However, it is likely that information will be available at different points in time, so that it would be better if further decision points were available in the future when information could be revealed.

The least regret solution that is identified is also technically checked *ex post* for robustness against the relevant security criteria. If the incumbent security criteria are not met, an analysis is carried out as to assess the techno-economic impact of changing the solution to comply with regulation, including reliability implications of not meeting the standards. If the economic implications of not meeting the security criteria outweigh the least regret cost of reinforcement, then a new relevant investment is selected. On the other hand, if the cost of reinforcement to meet the security criteria outweighs the economic implications of a reliability decrease (which is assessed offline through different analysis), NGET would seek a derogation from Ofgem. This option to diverge from the security standards process is allowed as from Condition C17 of National Grid's Licence. It appears that a similar approach could be applied to ENWL's DSR vs network reinforcement problem that is being analysed here. In particular, the potential implications of not meeting the P2/6 Engineering Recommendations (derogations are already sought today in certain cases) when carrying out alternative interventions to expand capacity (and specifically through post-contingency DSR) could be analysed as in the CBA models that Ofgem uses for ENWL's and the whole society's cash flows, so as to make a more informed decision or to carry out a more informed discussion with the Regulator. On the other hand, this requires network specific reliability analysis and an agreement on the cost of reliability (for instance for customer interruptions and customer minutes lost) to be coupled to the techno-economic assessment considered here.

While the analysis aims at producing investment strategies for the entire RIIO-T1 period, NGET proposes to review them every year through consultation with the relevant stakeholders, so that some investments might be brought forward while others might be delayed with respect to the initial plan also in the light of new information available. This "rolling decision horizon" would also partly address the concern that the least regret analysis would be based with respect to a given point in time in the future when information is assumed to be revealed. The benefit of this year-by-year approach to least regret analysis is thus that continuous changes in the expected scenarios are always included.

Related to this, taking account of the staging of different technical options is an intrinsic feature of NGET's strategies under consideration, as it allows decisions to be changed while specific work is in progress if different scenarios materialise with time. In particular, since the physical build of a new asset usually goes through successive and possibly lengthy stages (scoping, optioneering, pre-construction and construction works, etc), these stages are explicitly accounted for so that regret-minimising alternatives are assessed at each different stage while more information is revealed, thus also minimising the risk of asset stranding. Hence, when putting forward a set of technically available options these are not limited to those corresponding to minimum cost designs but also include staged investment.[1] For example, in the hypothetical case of an intervention subject to high uncertainty where doing nothing is the minimum cost solution in one scenario and an expensive reinforcement is the optimal solution in a different scenario, an intermediate option might also be considered to complete an incremental reinforcement. This would in fact allow completion of the full reinforcement if the relevant scenario materializes with time, as well as minimization of the regret in the case the do-nothing scenario were to occur. In particular, it may be the case that a particular transmission strategy selected in a previous year does not prove to be the least regret option any longer identified in the current year. In this situation, a decision as to whether to move on or not is made based on a comparison between the cost of cancellation and the cost of completion of the solution previously identified, including potential intermediate options to slow or delay completion to reduce potential future regret.

Ofgem support NGET's least-worst regret approach and the rolling basis of the scenario update as "appropriate, sufficiently proactive, prudent and flexible" for inherent risks associated with large investments with long lead times. Our view is also that this approach is sound as it mimics the behaviour of a decision maker that takes into account the best information about potential futures which is available at the time.

Although the NGET document never explicitly mentions the word "real options", the approach is grounded in the "RO thinking" rationale, particularly in light of the openness to considering rethinking of decisions with time in a receding horizon manner. In this sense, although the modelling details are not strictly speaking RO, the consideration of new information and the flexibility to respond to this brings value relative to traditional studies. However, rather than considering the expected value of the possible options in order to make decisions and in particular to delay or stage potential investments, as in classical RO modelling with a risk-neutral decision maker, NGET adopts a least regret approach that is more typical of a risk averse decision maker.

3.1.2. The Joint Regulator Group document and the "Spackman" approach

In the NDP, NGET adopts the "Spackman approach", promoted by the Joint Regulators Group (JRG) in its Technical Paper "Discounting for CBAs involving private investment, but public benefit" [JRC, 2011], to discount costs and benefits of potential network solutions. This model applies to a firm that finances an investment whose benefits mainly accrue to consumers or the wider public. More specifically, in the Spackman approach, whose adoption from NGET is also supported by Ofgem, all costs (including financing costs based on WACC, which is typically estimated by regulators for the specific firm in the respective regulated market and which in the specific case is 6.25%) and benefits are discounted at HM Treasury's social time preference rate – STPR (which is effectively given by the HM Treasury Green Book and which in the specific case is 3.5%), with the argument that the actual underlying systematic risk[2] is likely to be negligible. More specifically, the Spackman process comprises of two steps:

1. Convert investment costs into annual costs using the WACC, and add these to the annual cash flows[3] (an appropriate time profile is needed at this stage, for instance assuming a flat annuity).

[1] It is worth mentioning here that in our model proposed below staged investment can readily be carried out by explicitly considering different strategies and investment trigger points for sequential investment (as opposed to a larger lumped investment, for instance).

[2] Although a controversial term in the regulatory documentation that has been reviewed, "systematic risk" is in general meant here as risk that cannot be hedged against through market diversification. In terms of network investment business, it can be assumed that most risk is systematic and therefore this can be significant.

[3] There may be specific cases when private financing costs are funded upfront by the public sector and so may not have to be added.

2. Use the STPR to discount all annual costs and benefits.

This approach originates from the principal–agent model that describes the relationships between parties involved in public policy (in this case the Government could be considered the principal agent, while other public or private parties are "other" agents). In this framework, private investments in projects driven by public policies are risky and as such require a risk premium or compensation which can be seen as part of the social cost of the project. Therefore, private costs should be priced as other future policy costs (and benefits) when calculating the NPV, on top of the risk valuation based on the private sector cost of capital. Hence, the approach ensures that the investments financing costs are adequately reflected in the CBA according to the relevant risk profile. It is worth pointing out that in the RIIO-ED1 CBA templates for DNOs Ofgem applies a similar approach, whereby the WACC is used for investment discounting (annuitisation) and then a lower discount rate (of "social" discount rate type) is applied to all cost and benefit cash flows including annuitised investment. Alternatively, the WACC could be adopted in all costs and benefits. This case is also considered as one of the possible alternatives (but not the favoured one) by the Joint Regulators Group, particularly when systematic risk (that is in general the most critical and challenging component to factor in a CBA) is likely to be significant so that using the STPR, which ignores systematic risk, might not be appropriate. It is also interesting to point out that none of the regulators adopts an option that at first glance might also appear appropriate of discounting some CBA elements at the relevant WACC and others at the STPR, depending on their likely systematic risk. As argued in [JRC, 2011] this might be because of the complexity of assessing the systematic risk of individual elements of a CBA.

3.2. ENWL's Real Options "strawman" spreadsheet example

We analysed the latest numerical example on RO modelling provided to us, contained in the Excel workbook "Numerical options example v4", also taking into account the general information available in the Ofgem CBA templates that were provided before with respect to sample cases of C2C intervention valuations.

The example spreadsheet is quite comprehensive and, as already discussed at the November workshop, reflects correctly the general RO thinking that is under analysis in this work, as well as the general rationale of the models that will be developed by the University in the next year for the C2C project. The example provided covers different potential intervention strategies (based on conventional reinforcement, DSR, and a mix of them) over a timescale up to 2023, and across five scenarios, including a "most likely" central one.

While as it will be seen in the next Section there will be differences in the RO model that we would like to propose, particularly in terms of modelling of uncertainties and the practical spreadsheet implementation, it is possible to say that there are no fundamental divergences between the RO thinking that ENWL has had so far and ours.

There are a number of points that we'd like to highlight in terms of the ENWL spreadsheet, mostly in terms of clarifications.

- Timescale: the worksheet currently covers up to 2023 (which was an arbitrary assumption to build up a working example), and it is appropriate to ask until what point in the future the model should go. While Ofgem indicates to extend the analysis over 45 years, we believe that eventually it is challenging to represent the future and therefore any planning horizon in a meaningful way beyond around 10-20 years, particularly if higher discount rates might be adopted in the future (so that the discounted cash flows might count much less than close values). For instance, peak demand forecasts are currently to end 2031, i.e., end of RIIO-ED2 (17 years ahead), so this might be considered a reasonable timeframe too. On the other hand, if high cost interventions were to be put forward towards the end of the planning horizon without considering the new asset's lifetime after the intervention, this might create biases relative to other "cheaper" operational solutions. Although not critical, particularly if relatively long timeframes and discount rates are used, one way to get around this is to estimate the salvage costs of all the potential assets at the end of the selected time horizon, for which informative assumptions about the company's asset depreciation model need to be put forward by technical experts[4]. However, generally speaking sensitivity analysis should be performed through the developed model to check the influence of selection of different investment timescales, also considering the shapes and the length of the relevant scenarios available. Hence, the tool should be flexible enough to consider variable timeframes.

- Scenarios: the 5 demand scenarios could obviously be extended to more. However, there is always a compromise between number of scenarios (as well as intervention strategies, as elaborated further below) and computational burden, also considering the value that additional scenarios might bring. In general, it is advised that the scenarios themselves come from expert input and considering the in-house expertise and other relevant projects from ENWL. Also, there may be consideration for always including "extreme" scenarios such as "optimistic" and "pessimistic" (the latter particularly in the case the investment decision is to be stress-tested by robust optimization, as explored further below). As for the new tool to be developed, it should be flexible enough so as to accommodate a variable number of scenarios as selected from the user (up to a reasonable upper bound that could be 10 or 15, for instance). The idea of considering a central scenario and others with lower probability weight so that the overall occurrence sums up to 100% is certainly valuable and in line with our thinking. Also, this does not preclude the additional inclusion of a least regret analysis whereby probability weights would not be used (in fact, this would correspond to a case of "robust optimization" or minimisation of the possible worst case losses, whereby the worst case situation may want to be avoided regardless of the probability of occurrence). Scenarios and weights should come as inputs from outside the model based on external considerations and expert input, and some of them should also consider extreme events and possible divergences in the longer term particularly for least regret analysis. Probability weighting of scenarios is in line with classical real options approaches and also with the one we will propose here. It is also true that it diverges from the least regret approach undertaken by National Grid as discussed above, which is fundamentally a robust optimization against worst case scenario. Nevertheless, probability weighting could be used also to serve least regret analysis, for instance by pointing out that the scenario driving the robust decision is extremely unlikely, so that probability weighting of the worst outcomes could be used to re-rank the solutions identified according to a pure least regret analysis.
- Interventions and strategies: a "strategy" is a predefined set of different interventions carried out at different tipping points. Strategies should (just as scenarios) come from outside and be based on expert input. While certain interventions might be triggered (and then the relevant "triggering" times determined) internally to the RO process/tool, it is important that an adequate range of possible interventions and strategies are incorporated in the model to cope with the different scenarios. For instance, in the considered example the two high level intervention strategies are (i) conventional

[4] Currently, ENWL depreciate everything to nil, and even if redeploying a refurbished asset the finance treatment is at nil value, hence particular attention must be paid to any such assumptions on salvage value. However, as mentioned it is likely that discounting in the long term would be such that salvage value assumptions (which have been considered here for the principle of carrying out like for like comparison of strategies and in line with the levelised cost analysis approach which is a well established technique in engineering economics) might not make a big difference in any case. Obviously, these need to be treated according to the incumbent regulatory framework, and in this case the investment tool to be developed can be made flexible so as to incorporate salvage value only if needed.

reinforcement and (ii) a fixed volume of DSR for as long as possible before reinforcing (if still required). While these are very sound, there is potential to include other strategies to be purposely devised though expert consideration, such as reinforcing even if DSR is available, or increasing DSR with time as required. In fact, these further strategies could improve the overall decision flexibility and thus give a better view of the possible options particularly in terms of staging decisions. Our proposal entails a focus on assessing different potential strategies across the available scenarios, so that a proper ex ante definition of the potential intervention and strategies is critical, including multiple reinforcements and with economy of scale considerations. This highlights the importance of defining carefully the "design options" discussed above, which always requires external engineering input into the techno-economic assessment.

- DSR volume, availability, and price: as a working assumption, currently one single block of DSR is assumed to be available regardless of the volume actually needed. This could be considered as a possible strategy but it is likely not to be optimal, so that other options should be considered such as DSR volumes to be contracted changing with the projected demand, possibly with a certain predefined margin. Another point to be highlighted is the need to identify what is the maximum level of DSR that can be expected to be contracted at any time (for instance, 30% of the peak demand), as this will directly impact on the physical threshold point for reinforcement. This will likely be network specific and needs to come from real customer information. In addition, short term random uncertainty in the possibility of contracted demand being unavailable when needed should also be considered (it will be in the model we will propose), as it might have implications on the DSR margin to contract beforehand. Finally, considerations for DSR contract prices to change with time and to be subject to uncertainty should also be added, as it is likely that the marginal cost of increasing DSR volumes over different years will be higher once the easier customers to contract are saturated.

- Input data, parameters, and components of the CBA: it is critically important that the base set of data that will be used in the study is agreed before the studies are carried out, as from our initial calculations the results may be very sensitive to the input data and assumptions. For instance, including losses, emissions, and reliability indices in the CBA could change substantially the outcome of the decision on the best strategy to adopt. Sensitivity studies are in any case highly recommended to test the possible change in outcomes as the base data is varied. Even if not directly incorporated in the CBA cash flow studies, certain indices should be considered perhaps as risk indicators, such as for instance possibility of not meeting the peak demand and so on, since this might have a detrimental impact on the future use of DSR as well as on reputation.

- Small scale uncertainty: uncertainty in input data and smaller scale variations (at a "micro-level" with respect to the scenario "macro-level"), currently not included in the example, should be discussed and input in the study. We will do this explicitly in our proposed model, partly as a model of noise relative to base projections. This might include variations in expected peak demand, possible variations in future investment costs and DSR contracting costs, and so on.

- Lead times: this is a relevant issue and should be included in all features of the model, also specific flexibility with respect to the investment stages during the lead period itself such as abandonment or mothballing. The 3 years and 1 year lead times for reinforcement and DSR, respectively, appear appropriate for illustrative purposes but the information should be as accurate as possible, particularly for the longer lead times.

- Reinforcement threshold: this is another key point related to lead times, as potential thresholds to be met are to be defined based on projections of demand or forecast. A possible idea is to use as the "trigger point" the transition to a certain Load Index (LI) band [Ofgem, 2013], representing the asset loading percentage at peak time and in this case the exceeding duration[5] of maximum loading (for instance, the "LI5" band corresponds to peak demand exceeding the asset firm capacity for more than 9 hours per year). The peak demand "triggering" rule might also be extended to including a second aspect relevant to the "slope" of the demand trajectory (observing the trend over the last year or few years). It would be preferable to consider the threshold in MVA rather than MW. This will likely require specific assumptions to be made on power factors. Further considerations are needed in this respect, also depending on ENWL's current design approaches.

- Consideration on the number of future demand scenarios that a given intervention would cover: while the analysis carried out in the spreadsheet is appropriate, the approach that we will propose will be different in terms of point of view. In fact, as discussed further below, the outcome of the RO model will be a "real-time" indication on the strategy to be carried out with a receding horizon. Hence, effectively the decision will be updated continuously, year by year. It is also worth mentioning that the RO tool would be such that the suggested intervention will always be able to meet the anticipated demand acceptably (independently of the long-term scenarios that are considered) as the lead time for the proposed optimal intervention is always explicitly taken into account in the "spot" investment decision.

- Discount rates and cost category of interventions: In the example, a pre-WACC value as in Ofgem's CBA template was considered as a working assumption. A more complex rule closer to the Spackman approach (for discounting of costs and benefits for assets privately financed but with benefits socially accrued) is actually applied in Ofgem's suggested calculations, as mentioned above. This implies that all costs are discounted in the same way and somehow belong to the same category. While this seems appropriate for the time being, it is likely that: (i) the regulatory position could enforce for instance the Spackman approach; and (ii) a different approach to discounting might be taken besides the suggested CBA framework and for internal strategic assessment, for instance if the discount factor were used as an equivalent risk metric, e.g., to take into account a certain risk premium associated to specific interventions (such as DSR) that might be considered riskier than conventional ones (such as asset reinforcement), as also discussed in Section 4.

- Identification of optionality from DSR interventions: towards the end of the worksheet, an analysis of the optionality that DSR could provide by allowing switching from interventions appropriate in higher demand scenarios to those more appropriate in lower demand scenarios is considered, so that even if DSR may not the best option in any perfect foresight scenario, it could still be the best alternative against uncertain demand. While there are appear to be no flaws in the analysis as conducted, it is worth highlighting again that in the approach that will be proposed later this aspect is automatically taken into account when analyzing individual strategies. In fact, at each receding horizon analysis point (every year, for instance), the *option value* of each strategy (and the optimal one in particular) can be calculated as simply the difference in values (according to the relevant metric) between that strategy and a business-as-usual intervention defined as the *baseline*.

4. A novel multi-layer receding horizon approach proposed for RO valuation of flexible distribution network investment with DSR

[5] In order to take into account exceeding capacity duration, assumptions on the peak load profile might be needed too.

4.1. The multi-layer spreadsheet model

4.1.1. General tool architecture

In light of the complexity and detail of the RO challenge faced by ENWL and based on the discussions carried out above in this document, we now describe a novel layered approach which aims to combine the best available approaches to RO valuation of flexible investment strategies as appropriate for this particular application and, in particular, its straightforward implementation in an Excel spreadsheet that could be reusable. We highlight the methodologies used and their assumptions, strengths, and limitations in order to give an understanding of the ways in which the model may be used and also adapted or extended as necessary.

The fundamental idea behind our proposal is to design a RO spreadsheet/workbook tool that relies on a multi-layer hierarchical approach to uncertainty modelling and decision making. In order to do so, we take the best RO approaches that are suitable for the purposes of the analysis and we combine them at the relevant levels, namely, we consider exhaustive search for strategy comparison, scenario based analysis for long term uncertainty modelling, and Monte Carlo simulations for short term uncertainty modelling.

The modeling layers correspond to physical locations in an Excel spreadsheet. Each worksheet corresponds to a strategy, and there is one additional worksheet (or it could be more) containing all input data and outputs. Each worksheet (say, the worksheet for Strategy X) contains all scenarios and their combinations, which are then analysed through Monte Carlo simulation under the application of Strategy X. More specifically, the following modelling layers are conceived:

1) Layer 1: *Strategy layer*. There will be as many worksheets as strategies, whereby a *strategy* is a predefined set of interventions that may take place at “tipping points” following certain rules related to the demand scenarios in Layer 2. This layer corresponds to a worksheet “Strategy X” (with X going from 1 to the number of design strategies D that are considered in the study), which contains all scenarios. As discussed below, it is important that experts define *ex ante* the strategies to consider in the analysis and their relevant features.

2) Layer 2: *Macro level scenario layer*. This layer models long term uncertainties (ie. the peak demand trajectory across future years), which as discussed below we consider is best modelled using scenarios (as in the current “strawman” example). This layer corresponds to each “Scenario Y” of the “macro level” long-term scenarios considered in the analysis. All scenarios are modelled within each strategy, so that each worksheet “Strategy X” contains each and all scenarios “Scenario Y”, with Y from 1 to the number of long-term scenarios S.

3) Layer 3: *Micro level simulation layer*. This layer corresponds to micro uncertainties and small scale variations occurring within each scenario, and as discussed below we suggest modelling these uncertainties based on simulation in a Monte Carlo context. In terms of implementation, for each worksheet “Strategy X” and each and all scenarios “Scenario Y” within the worksheet, Monte Carlo simulations (with a number of simulations N in the order of 1000) are run to create a probability distribution corresponding to each “Scenario Y” and “Strategy X”. Then, as discussed below, these scenario probability distributions can be opportunely combined across scenarios to yield an overall probability distribution for the “Strategy X”, based on which suitable summary metrics are calculated, the different strategies (the results from the different worksheets) compared, and the best one selected according to the preferred criteria.

A schematic representation of how the Excel workbook tool might look like is provided in Table 2, also including a specific worksheet containing the general controls, input data, summary of the results according to different metrics, and optimal control strategy to be selected based on the information of the strategy worksheets and relevant criterion. Specific details on the models to populate the tool are provided in the sequel.

Table 2. Schematics of possible workbook architecture of the proposed RO tool

Worksheet "Strategy 1"		Worksheet "Strategy 2"		Worksheet "Strategy X"		Worksheet "Strategy D"		Main
Scenario 1	Monte Carlo simulation	Scenario 1	Monte Carlo simulation	Scenario 1	Monte Carlo simulation	Scenario 1	Monte Carlo simulation	
	1		1		1		1	
	2		2		2		2	
	---		---		---		---	
	N		N		N		N	
Scenario 1 summary		Scenario 1 summary		Scenario 1 summary		Scenario 1 summary		
Scenario 2	Monte Carlo simulation	Scenario 2	Monte Carlo simulation	Scenario 2	Monte Carlo simulation	Scenario 2	Monte Carlo simulation	
	1		1		1		1	
	2		2		2		2	
	---		---		---		---	
	N		N		N		N	
Scenario 2 summary		Scenario 2 summary		Scenario 2 summary		Scenario 2 summary		
Scenario Y	Monte Carlo simulation	Scenario Y	Monte Carlo simulation	Scenario Y	Monte Carlo simulation	Scenario Y	Monte Carlo simulation	
	1		1		1		1	
	2		2		2		2	
	---		---		---		---	
	N		N		N		N	
Scenario Y summary		Scenario Y summary		Scenario Y summary		Scenario Y summary		
Scenario S	Monte Carlo simulation	Scenario S	Monte Carlo simulation	Scenario S	Monte Carlo simulation	Scenario S	Monte Carlo simulation	
	1		1		1		1	
	2		2		2		2	
	---		---		---		---	
	N		N		N		N	
Scenario S summary		Scenario S summary		Scenario S summary		Scenario S summary		
Strategy 1 summary		Strategy 2 summary		Strategy X summary		Strategy D summary		Overall summary and optimal strategy

4.1.2. Layer 2: Long-term scenarios analysis for macro factors (demand growth) and receding horizon

For the sake of clarity, layers 2 and 3 will be discussed here before layer 1.

Regarding layer 2, we propose the selection of a manageable number of scenarios for the most important uncertain variable, namely *peak demand growth*. This approach allows the decision maker to clearly and directly specify the model of future uncertainty in demand, the most significant driver of network investment strategy, based on expert input rather than on arguable stochastic processes. It is recommended that, as far as possible, scenarios are chosen in such a way that they are perceived to be *equally likely* as futures. However, we recognise that the likelihood of certain scenarios will be driven by exogenous factors (such as future economic and technological developments) outside the decision maker's control. Where any scenario is perceived as being significantly more or less likely than others, we therefore recommend that a specific and appropriate *probability weight* is assigned to that scenario, and that the probability weights of all other scenarios be adjusted to account for this (this adjustment can be automated in a spreadsheet). As an example, a central scenario may be up-weighted and extreme scenarios may be down-weighted, while all other scenarios remain equally probable (this was already done in the ENWL's "strawman" example). However, this is not strictly necessary, and any combination of probability weights may be used, provided that they sum to 100%.

Of course, this recommended approach implicitly assumes that the given scenarios are the only possible futures. While this may be an acceptable approximation, it should also be clearly recognised that these scenarios are all designed to start from the present time and, as such, it may not make sense to reuse them in, say, three year's time when new information may have become available. Further, we assume that the scenarios chosen do not include any 'nodes' or 'branching points' at future times, but are single paths. This assumption is made for simplicity of modelling as we consider it to be superior to, for example, lattice methods, differential equation based methods, and simulation methods in terms of being clearly justifiable and understandable: the scenario approach does not require any particular assumption to be made about the random dynamics of peak demand process over time, which is instead required in all the other abovementioned approaches and is, in our view, difficult to justify. Indeed, it would be possible to elaborate within the scenario approach by including internal branching points and this could be done, but in our view the appropriate choice of scenario weights becomes in that case significantly more challenging.

This recommended scenario approach means that we do not model optimisation decisions taken at future times since we have no model for the scenarios that would be considered at future times (nor their weights) and hence cannot mathematically model the future decision making process. This assumption impacts the type of strategies which may be valued using this model: we are limited to considering *strategies consisting of a set of interventions, which take place at certain trigger points*. As a result our analysis aims only to approximate the optimal strategy, since the optimal strategy may in general require future optimisation decisions which are outside the scope of modelling and implementation in a relatively simple spreadsheet tool. However, it is an established principle in control engineering that such approximately optimal strategies can perform well in the context of a *receding horizon* approach, whereby only the first step (for example, the first year) of the approximately optimal path is carried out before the model is re-evaluated in light of the new state of the world and new information. Then, the first step of the re-evaluated strategy is carried out (instead of the second step of the original strategy), and so on. The receding horizon principle is exemplified in Table 3.

Table 3. Schematics of the receding horizon approach

Strategy number	Years from today (planning)														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	…
1	*															
2		*														
3			*													
4				*												
5					*											

…

Strategy Implementation	*	*	*	*	*	…

Clearly the actions which are eventually implemented in this receding horizon approach do not necessarily correspond to *any* strategy (again, described using interventions and tipping points) that could be derived today. Indeed, only the first part of the eventually implemented actions can be calculated today; the second part will be calculated in one year's time, and so on. Hence this approach, besides being easily implementable and understandable, does actually mimic the "on the spot" decision making process that a company has to carry out. In addition, the continual process of strategy re-evaluation does address the fact that our choice of strategies is limited, as it allows for optimisation to be done in later years – this was the major weakness identified in our scenario-based approach, and the receding horizon is a good way to make up for it. The difference with respect to more sophisticated modelling that would be much harder to implement is simply that this optimisation is done in real time rather than being modelled mathematically beforehand. Additionally, the continual re-evaluation of strategies means that the question of choosing a planning period (or equivalently planning horizon) becomes less significant as the end of the planning period is continually moved back one step, thus taking out further uncertainty in "modelling the future" and particularly the distant future, provided that adequate residual or salvage values can be assigned to those assets which remain usable at the end of the planning period. The *effective* planning horizon for this receding horizon approach is therefore infinite, since the receding horizon approach continues indefinitely. We recommend sense checking of the set of scenarios and their weights by the decision maker each time the model is reused: we understand that this is also ENWL's view.

A summary of the main considerations relevant to Layer 1 is reported in Table 4.

Table 4. Summary critical evaluation of Layer 1

Layer 1 Feature	Advantages	Limitations
Use of Scenarios	Simple, direct, interpretable modeling of macro factors (peak load growth)	Cannot model *future* scenario analysis, so strategies must be sets of interventions defined by physical 'tipping points'
	Fine detailed modeling of micro factors is done *within* scenarios, then combined *across* scenarios in a weighted or robust fashion	Scenarios and weights should be re-derived by an expert on each future reuse of the model (this would apply to all reusable models though)
Receding horizon approach	Receding horizon approach makes particular choice of planning horizon less important	Only *approximately* optimal strategies are derived, to be updated each year ('receding horizon approach')
Strategy analysis	Allows for least-worst regret analysis	Requires one worksheet (containing all macro scenarios) per strategy considered

In addition to its clarity, the scenario approach makes it possible to include a wide range of criteria and metrics for the decision maker. As discussed above, the default RO criterion is average cost (or benefit). However, in the scenario approach it is also straightforward to calculate and visualize the full distribution of future costs, and not just its average value: this makes it possible, for example, to supplement the mean value with financial risk indicators for the costs (VaR, CvaR, etc). In this way, scenario analysis provides a simple approach to accounting for risk. Additionally, scenario analysis makes possible robust analysis, for instance, least-worst regret analysis in which a measure of "regret" is allocated to each possible future by subtracting the lowest actual cost under other strategies, as also described in Section 4.1.4. This is analogous to the above umbrella-carrying example. In light of the above, we recommend that a multi-criteria approach is taken to decision making in the RO model for network investment (see Section 4.1.4 for details of our proposal), in which several criteria (including average cost, risk measures, and regret) are all evaluated separately, and then these results are considered side by side in decision making, as mentioned above.

4.1.3. Layer 3: Short-term Monte Carlo analysis for micro factors (e.g., DSR availability and contract cost, peak demand noise)

Having specified the set of scenarios for demand growth, we now recommend that an alternative approach is taken *within* each given scenario. For the purposes of discussion, we therefore now suppose that we have selected just one of the scenarios and we will develop it further to account for uncertain variables which are significant at a more "micro" level: we will therefore make no further reference to alternative demand growth scenarios within this Section. More specifically, within each scenario we may lack "fine" knowledge about the real-time performance of DSR, which may be lower than expected either because of an insufficient number of contracted customers or because not enough DSR load is online for disconnection when needed; similarly, we may lack knowledge over contract price expectations from DSR customers, or there can be noise on top of expected peak demand.

Given the "micro" classification of these uncertain variables, and possibly also given a lack of knowledge about their behaviour resulting for example from a lack of comparable historic data, we do not recommend the use of probability weighted scenarios for Layer 2 nor of methods which rely on specifying random dynamics, such as lattice or differential equation methods. Insofar as there are additional expert insights, there is much scope to implement these *within* the Layer 2 approach. We therefore recommend that these variables are modelled straightforwardly by simulation as uncertain physical states which evolve randomly through time within the context of Monte Carlo analysis. More specifically, first an average level is specified for each uncertain variable (which is analogous to the long-term scenarios trajectory of layer 2), and then a distribution is chosen for the randomly distributed errors around this average level. This approach may also be interpreted as a randomized version of scenario analysis. We may suppose that these uncertain variables are independently regenerated each year so that there is no correlation between for instance the DSR availabilities in different years, or the DSR payments in different years. At the opposite extreme within this Monte Carlo approach, we may model an 'underlying' level that is unknown today but nevertheless which will be consistent from year to year (for example, either 80%, or 90% or 100% of DSR availability; this introduces perfect correlation between the availabilities in different years). These two approaches may also be combined (i.e., consistent but unknown mean level, plus independent yearly deviations) to create a partial correlation between the values of these uncertain variables.

If the choice of error distribution as described above is held constant over future years then this makes the implicit assumption that the patterns of errors are unchanged from year to year; an alternative, more detailed approach would be to gradually vary the distributions further out in time within the model. As an example, we may wish to model a gradual increasing or decreasing trend in DSR costs (this approach was taken in modelling costs of distributed generation in [Hoff et al, 1996], or a gradually improving scenario range to reflect improved control and implementation of DSR as time proceeds, and so forth. Alternatively or additionally, the trend in DSR costs could also be made to track the trend in peak demand to reflect contract prices, for instance in the case of increasing volume of DSR needed to counteract load growth which would be available at increasing marginal cost (it is expected that the more available and marginally "cheaper" customers would be contracted first). On the other hand, if the DSR market were to become competitive, contract prices might even go down or in general fluctuate over years. This can all be readily modelled subject to expert feedback on the assumptions. On the other hand, the simulation approach described for layer 3 has the advantages of being less specific than the scenario approach indicated for layer 2, and hence less dependent on both expert input and the availability of related historical data (although the model parameters can of course be fine-tuned later while more evidence of process trajectories may be available).

A summary of the main considerations relevant to Layer 3 modelling is shown in Table 5.

Table 5. Summary critical evaluation of Layer 3

Layer 3 Feature	Advantages	Limitations
Micro variables	Multiple micro variables considered within each macro scenario offering simple, direct, interpretable modeling	Large number (>10^3) of Monte Carlo simulations (i.e., rows) needed per worksheet – this number increases with the number of micro variables
Monte Carlo approach	Each Monte Carlo simulation (or 'micro scenario') includes modelling of lead times, peak load projections, etc., so can be sense checked as a (retrospective) CBA	Monte Carlo results (i.e., after averaging) are *slightly* different every time so repeatability should be checked and , if necessary, the number of Monte Carlo simulations increased accordingly
Strategy analysis	Full *per-scenario* cost and physical risk distributions obtained for each strategy	One Excel worksheet per strategy means that strategies should be hand picked

A summary of the main characteristics of layers 2 and 3 is reported in Table 6. Based on this, we can proceed with the successive analysis which refers to summarizing results at a strategy level.

Table 6. Summary of Layers 2 and 3

Layer	Uncertainty model	Variation	Expert input	Output
2	Probability weighted scenarios	Peak load growth (pathway)	- Choice of peak load pathways - Probability weights	- Per-strategy weighted cost distribution - Per-strategy weighted risk metrics - Least-worst regret analysis
3	Monte Carlo simulations	- DSR costs - Peak load adjustments (annual volatility) - DSR unavailability	- DSR cost distributions - Peak load adjustment and DSR unavailability distributions	- Per-scenario cost distribution - Per-scenario physical risk distributions

4.1.4. Layer 1: Strategy modeling

Each strategy is specified by identifying all of its specifications, an example of which is described in Table 7. Specific description requirements need to be tailor made based on expert techno-economic input.

Table 7. Example of specifications of a strategy

Strategy Feature	Description
Peak demand projection	Specification of the method used to project an expected peak demand level at various horizons from today (e.g., 1 year for DSR contracts, 3 years for reinforcement); may e.g. be taken from the peak demand change scenarios but could in principle be different
List of states and lead times	Takes account of lead times, e.g., reinforcement states may be coded as 3/2/1/0, respectively denoting the number of years until reinforcement will be complete. DSR state may e.g. record the total size of DSR contracts being renewed/negotiated for the coming year
State transition rule	For each possible state above, a decision rule is specified for progression to the next state (e.g., reinforcement state 2 automatically moves to state 1 as the reinforcement work nears completion; or reinforcement work starts when 3-year projected demand exceeds total capacity net of DSR)
Costs/benefits	Full list of all costs and benefits (e.g., investment cost, DSR cost, salvage values, compensation costs), possibly depending on the current states, sufficient to carry out a retrospective CBA in each micro scenario
Discount rate	As suggested in Section 2.5.2, different discount rates or risk premia might apply to different strategies (or to specific costs/benefits within a strategy). However, the scenarios in Layer 2 and the detailed simulation present in Layer 3 is sufficient to account for risk and we do not recommend using the “crude” technique of modifying the discount rate; instead, for clarity and interpretability it is recommended that a single discount rate be kept within each workbook

Since strategies typically involve tipping points which need to be foreseen years in advance due to the significant lead times potentially required for capital investment, this requires specifying the basis on which demand projections are made. Clearly an exhaustive list of all possible strategies would be extremely lengthy, and we recommend creating one Excel worksheet per strategy, so an expert should be involved in identifying a manageable number of candidate best strategies for examination by the model. As an example, we may consider a strategy which aims to perform DSR for as long as possible then reinforce if necessary. In this case, a precise meaning must be given to 'as long as possible' – for example, the tipping point may be "when projected peak demand in three years is greater than 0.9 times current capacity", so as to leave a security margin, if wished. As a consequence we must also give a precise meaning to "projected demand" – for example, "demand is projected by assuming that the demand growth seen in the current year continues unchanged over the next three years". A sensitivity analysis is of course possible whereby the demand projection method is varied in order to find the best balance between caution and expected cost: this analysis would be performed by varying *only* the demand projection method, and considering each variation to be a different strategy, then comparing.

With our chosen set of strategies precisely defined, for each strategy (corresponding to a worksheet) we may now consider a single macro scenario from Layer 2. The sequence of decisions taken by rational management over any set of micro variations around this macro scenario can then be calculated by applying the strategy, and the cost distribution of *this strategy* within *this macro scenario* may be calculated by Monte Carlo analysis in which a large number of micro-scenarios are simulated, and a retrospective CBA performed on each one to arrive at its net present cost. The empirical distribution of this set of simulated NPCs may be summarised in the form of a histogram, *for this macro scenario*, with appropriately sized bin widths. In this way we obtain one histogram per macro scenario, which may be combined as explained in the next paragraph. Before combining, however, we note that some additional metrics may be calculated at this stage. Firstly the average cost *of this strategy in this macro scenario,* minus the cost of the cheapest strategy *in this macro scenario,* may be used as a measure of 'regret' as discussed above; secondly, averages may also be taken of any physical stress indicators – e.g., the proportion of micro scenarios in which peak demand was not met.

We now repeat this process *for the same strategy* (the same worksheet) for *all scenarios* (all coded within the same strategy worksheet) and we create relevant distributions and histograms for each of them. It may be shown (using the Law of Total Probability) that these per-macro scenario distributions *for this strategy* may now be combined in a mathematically consistent way. Assuming that consistent bin widths have been used across the macro scenarios for the same strategy, taking weighted averages of the bin heights in the obvious way (using the macro scenario probability weights) then finally yields the estimated overall cost distribution *for this strategy, across all scenarios.* From this overall cost distribution, the overall average cost and financial risk metrics may be calculated (VaR, CVaR, etc.). Overall physical risk associated *to this strategy* may also be evaluated by taking the probability weighted average of any physical stress indicators calculated as described in the above paragraph. Finally, the worst regret *for this strategy* may be evaluated by simply identifying the scenario with the highest regret; it may be informative to also highlight how likely is the macro scenario which leads to the highest regret (and as mentioned earlier, potentially the corresponding probability weight can be applied too, so that the "least worst regret" criterion turns into the "minimax weighted regret" one).

4.1.5. Exhaustive search and optimal strategy identification based on the relevant metrics and decision models

Finally, having now calculated all of the desired metrics for one of our given strategies “Strategy X”, the same process is repeated for all strategies under consideration (all the worksheets). In this way all metrics (from average cost to risk-based measures) are in place for a full multi-criterion analysis of the set of candidate strategies and the final selection may be made by exhaustive search across all strategies (all worksheets) according to the relevant criterion selected.

In order to exemplify the overall procedure for optimal strategy selection through the proposed multi-layer tool, let us take for example the worksheet “Strategy 2” (Layer 1), evaluated across three macro scenarios “A”, “B”, and “C”, with characteristics as in Table 8.

Table 8. Characteristics of Strategy 2 in the example case

Macro scenario	A	B	C
Cost £M (ignoring micro uncertainty)	7	6	5
Micro uncertainties	Low	Med	High

The indications on micro uncertainties in Table 8 are visually reflected by the cost probability distribution functions (PDFs) from Monte Carlo simulations (Layer 3) as from Figure 1, where the variance of the data with respect to the expected values increases moving from Scenario A to Scenario C (Layer 2).

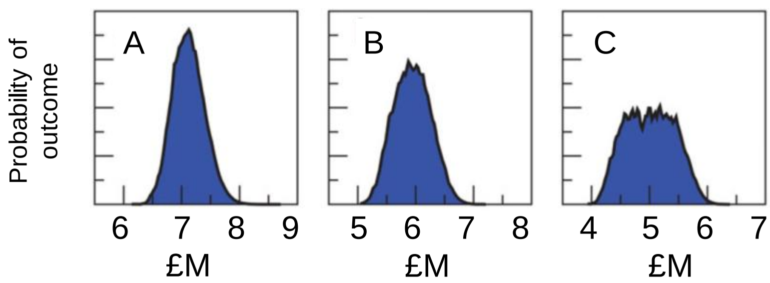

Figure 1. Monte Carlo based PDFs for the three example scenarios in Strategy 2

The macro scenario weights are then used to merge these PDFs for overall risk analysis and comparison against other strategies (Figure 2) in Layer 1, with Expected Cost and VaR values also indicated in Table 9.

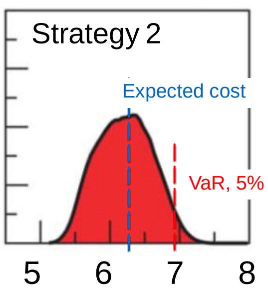

Figure 2. Monte Carlo summary for Strategy 2, Layer 1

Table 9. Summary of the metrics for strategy comparison in Layer 1

Strategy	1	2	3	…
Expected Cost £M	…	6.3	…	…
Value at 5% Risk £M	…	6.9	…	…

Different strategies can then be compared with each other based on their Expected Cost or other (risk) metric, in the specific case the VaR, as exemplified in Figure 3 for hypothetical Strategy 1 and Strategy 2.

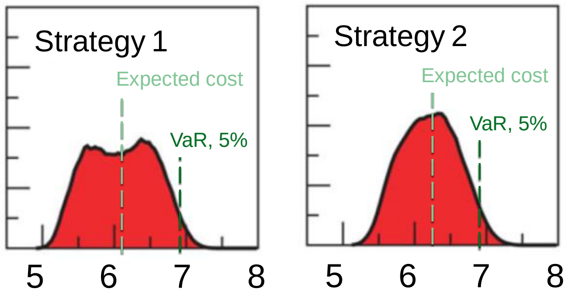

Figure 3. Visual comparison between Strategy 1 and Strategy 2

It is interesting to notice how Strategy 1 is characterized by lower expected cost but higher variance, while Strategy 2 has higher expected cost but lower variance. However, both PDFs are relatively symmetric and have approximately equal VaR. Hence, not much information could be drawn in terms of risk management from the VaR indicator, and in case the variance should be used as risk metric. Therefore, a risk-neutral decision maker might decide to focus on expected cost and therefore go for Strategy 1, while a risk-averse decision maker might opt for Strategy 2, if the variance was the risk metric. On the other hand, in the generic PDF shape examples shown in Figure 4, the skewed distributions (which one could imagine corresponding to as many strategies to be assessed in Layer 1) have approximately the same variance, but significantly different VaR values, so that the opposite situation might arise.

These simple examples thus illustrate the rationale behind our proposal of adopting multiple metrics and risk indicators when comparing different strategies, given the variability of PDF shapes that could arise from different strategies. Also, these examples illustrate the benefit of adopting a multi-layer approach with differentiation between macro (scenario-based, Layer 2) and micro (Monte Carlo-based, Layer 3) uncertainties which are then combined at the Layer 1 level to give a picture of the full distribution of costs, so that specific details in the interpretation of the risk metrics for each strategy can be explicitly checked.

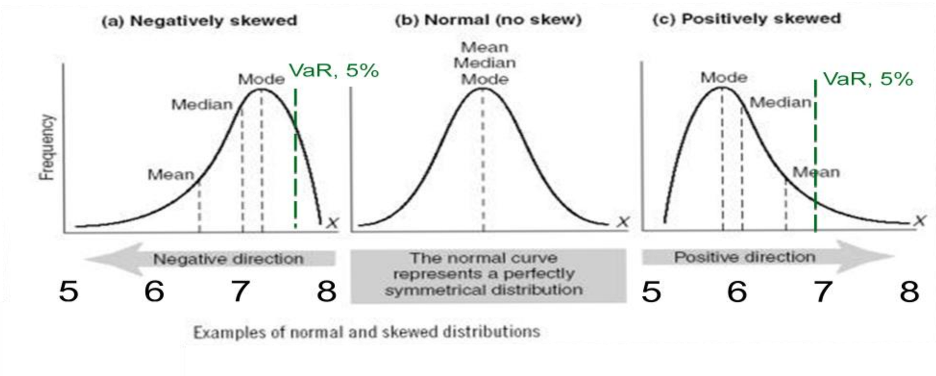

Figure 4. Visual comparison between example PDFs with different shapes

However, as mentioned a few times alternative approaches to decision making could be carried out, for instance based on *minimizing the worst regret*, as in National Grid's approach. More specifically, putting it in the context of our model this rule would

- Calculate for each Scenario S the "regret" experienced from each Strategy X:

Regret(X,S) = (Cost of Strategy X under Scenario S) *minus*

(cost of *Optimum Strategy* for Scenario S)

- Calculate for each Strategy X the "worst regret" that could possibly be experienced across all possible Scenarios S:

WorstRegret(X) = maximum{Regret(X,S)}

- Select the investment Strategy X* whose worst regret is the smallest (the "least worst regret")

Strategy X* -> min WorstRegret(X)

In the following illustrative example, starting from the cost of each strategy and each scenario (Table 10), it is possible to build the Regret matrix of Table 11, whereby the regrets in each scenario are calculated with respect to the best strategy (minimum cost) in that scenario. For instance, in Scenario A the regret for Strategy 1 is equal to 2 (difference between the cost in Strategy 1 and the cost in Strategy 2, which is the best strategy in Scenario A), the regret for Strategy B is zero since Strategy B is the best strategy for that scenario, and so forth. Therefore, the worst possible regret for each strategy across all scenarios occurs under Strategy 3, so that Strategy 3 is favourable according to "least worst regret" analysis. As mentioned earlier, other versions of this decision approach could also consider the relative weights of the different scenarios when building the relevant decision matrices (see for instance [Carpaneto et al, 2011b]).

Table 10. Strategy/Scenario cost matrix

Cost	**Scenario A**	**Scenario B**	**Scenario C**
Strategy 1	4	4	12
Strategy 2	2	3	8
Strategy 3	3	2	1
Strategy 4	3	4	12
Best strategy in scenario (and its *cost*)	2 (*2*)	3 (*2*)	3 (*1*)

Table 11. Regret matrix

Regret	**Scenario A**	**Scenario B**	**Scenario C**	***Worst regret***
Strategy 1	2	2	11	11
Strategy 2	0	1	7	*7*
Strategy 3	*1*	*0*	*0*	***1***
Strategy 4	1	2	11	11

4.1.6. Considerations on computational time

Computational time will of course be influenced by the number of scenarios considered in Layer 2, the number of micro variables considered in Layer 3, and the number of metrics used in the analysis. However, the greatest effect on computation time is expected to come from the Monte Carlo step. To test this, we ran a simple example Monte Carlo simulation in Excel. On a standard desktop computer (2.5 GHz, 4GB RAM) it took:

- <1 second to run 1,000 Monte Carlo simulations and the results were consistent and repeatable within a 3% tolerance;
- 25 seconds to run 10,000 Monte Carlo simulations and the results were consistent and repeatable within 1% tolerance.

Our proposal is based on the use of approximately 1,000 Monte Carlo simulations per worksheet, within which all scenarios are examined as discussed above. Given that the purpose of this tool is to provide a broad comparison of a number of strategies across a number of scenarios under a number of metrics, the example tolerance level we observed above with 1,000 simulations seems acceptable for an Excel tool for use on a standard computer. Repeatability can easily be checked by simply refreshing the spreadsheet, and we of course recommend that marginal results from any modelling exercise should always be investigated further in any case. We would also like to point out that we expect (although we have not tested this) that, if deemed necessary, the Monte Carlo process might be sped up by "coding" the simulations for instance in a VBA macro rather than using the standard Excel functions, albeit with a corresponding increase in the complexity.

4.2. Risk modelling: financial and physical risk measures

As widely discussed above, risk can be accounted for in different ways in our proposed tool, for instance by using different and appropriate discount rates for different interventions and over time, or by considering various "tail" measures at the level of scenario cost distributions and overall strategy cost distributions, such as for instance VaR and CVaR. The decision maker would therefore typically have at disposal a full set of information to undertake the best strategy, for instance based on average cost only (risk-neutral decision maker), by considering only the relevant risk measures (risk-averse decision maker), or by applying a mix of decision criteria (see also Section 4.1.5). For instance, a risk-averse decision maker could decide to go for a strategy for which the selected CVaR or the worst regret is minimum. A risk-neutral decision maker could go for a "classical" approach and pick the strategy that would minimise the average cost. Considering both at the same time would represent a multi-criteria approach, which could be reconducted to a single criteria approach for instance by deciding to weigh (in a way to define *ex ante*) the expected cost and the relevant risk measure so that an intermediate level of risk aversion could be achieved.

Other risk measures, physical rather than financial, could also be considered, for instance to take into account that different interventions provide different amounts of capacity, and DSR could potentially be less reliably available than a new asset (this has already been incorporated in our proposal through layer 3 short term uncertainty modelling). In particular, for instance a "flag" could be put any time (any Monte Carlo simulation run) that peak load might potentially not be met according to our layer 3 modelling in a given scenario and a given strategy. Then, exactly as for the financial indicators, probability distributions at the scenario level (layer 2) and the strategy level (layer 1) could be built, so as to identify and quantify under which conditions (scenarios) a given strategy might bring technical risk of not meeting the load or exceeding the asset limits.

Technical risk metrics might be represented for instance by Load Indices (LIs) [Ofgem, 2013], basically representing demand versus capacity and decreasing (in a scale of 1-5, from low to high) if demand falls or if capacity is increased. LIs could be easily incorporated in our simulation model and a probabilistic representation of risk through this measure could be readily provided. As suggested by Dr Shaw in one of our private communications, even if currently DNOs do not get a direct financial benefit from improving LIs, there exists a regulatory commitment that this measure should not exceed certain threshold measured in terms of overall number of LIs per year or overall weighted LI score. Hence, if incorporated in our simulations, our approach could truly be multi-criteria in the sense that it could provide a probabilistic representation of both financial and reliability performance of a given strategy, with also detailed breakdown by scenario so as to identify specific drivers for risk and potential interventions. Such LI risk measures (or similar measures such as Health Indices, HIs, which could significantly change following a capacity intervention) would also provide strategic indication on the performance of an asset portfolio, and in case it could be decided to include them or not into the analysis (thus changing the degree of risk aversion, as in the umbrella carrying example) depending on the available "headroom" available before being at risk.

Various sources of technical risk might also be taken into account, for instance based on ICT failures in operating NOP or DSR, which could either coded within our short-term uncertainty model or could come externally from reliability simulations specifically designed and run so as to be consistent with our RO approach (this latter case obviously require inputs and work outside the RO tool only, particularly if realistic assessment of expected energy not supplied were needed).

Financial value could then be assigned to technical risk metrics too, either because enforced by Regulation through penalties, or because set internally by the DNO, for instance to represent overall costs related to factors such as interruption or reputation costs in the event that capacity were insufficient to meet demand. In this way, we would fall back into purely financial, but risk inclusive, single objective optimization.

4.3. DSR "pricing"

Our model proposed and described above may also be used to examine the question of the correct price to pay for the option to get DSR at a future date. Since the model of DSR price is one of the features that must be specified for each strategy (see Table 7 above), it is an input to our modeling. In our simulation-based approach, the DSR contract structure could readily include separate payments for availability and utilization by including sufficient detail in the list of costs and benefits included in the strategy. Note that the DSR price may be specified to be stochastic, in which case it is the mean level (or trend) that is specified, together with a distribution for the deviations from this price. A set of new strategies may then be constructed, differing only in the model of DSR prices, and these new strategies compared in the normal way (taking into account their average cost, least-worst regret, etc). In this way, again by exhaustive search by running various sensitivities on the DSR price, an optimal price level could be reached according to the specific criterion considered. It could also be that different strategies (for instance, adopting DSR till a certain capacity threshold is met by load growth as opposed to immediate asset reinforcement) might become optimal under different DSR payments, which again could be identified through sensitivity studies (this aspect might be particularly relevant since, based on a number of test performed within the C2C project, optimal strategies may be quite sensitive to DSR contract prices). The approach undertaken in this way to DSR pricing is also multi-criterion, making it consistent with the overall approach to optimal strategy selection.

5. Concluding remarks

5.1. Summary of the work

In this report we have provided a comprehensive overview of the state of the art of Real Options analysis and risk assessment, with focus on applications to flexible network investment under uncertainty and with insights on the issues and opportunities that inclusion of DSR might bring. We have also reviewed current approaches that have been undertaken for decision making under uncertainty by National Grid in their Network Development Policy document and by ENWL in their "strawman" RO spreadsheet example. Finally, based on our expertise, experience and studies carried out during this work, and our understanding of ENWL's requirements as to a RO tool to be developed in Excel, we have proposed a novel methodology for RO assessment of network investment including DSR and described relevant worksheet architecture. This point is expanded on below.

5.2. Proposed RO model and tool architecture

We have proposed a novel multi-layer receding horizon approach, exemplified in a hierarchical spreadsheet implementation, to RO analysis of flexible network investment under uncertainty with specific inclusion of DSR.

The model is organised in terms of strategy (layer 1), long-term scenarios (layer 2) and short-term Monte Carlo simulations (layer 3), thus bringing together and deploying the optimal features of different RO approaches as fit for the purpose of this work.

The proposed tool can be flexibly adapted to take decisions on a regular basis (for instance, every year), and the underlying model features the upsides of the receding horizon approach successively deployed in the engineering applications of optimal control theory and also makes up at the same time for some limitations that implementation in a relatively simple tool brings.

Different metrics and decision criteria have been discussed and can be implemented in the tool, based on probabilistic representation of relevant random variables and allowing specific consideration for financial and physical risk analysis and hedging of different strategies to be considered.

Useful outputs of the proposed tool may include:

- Optimal investment strategy for the current year (decision time), to be reassessed with receding horizon every year in the light of the projected scenarios and estimated uncertainty.
- Ranking of the considered decision strategies based on the input intervention alternatives (the "design-and-time options") as from different criteria (expected value, expected value weighted with risk metrics, least worst regret, weighted least regret, VaR, CVaR).
- Detailed breakdown of the probabilistic distribution of costs of each strategy in each scenario, so that fully informed and transparent decisions can be made.

The tool can be applied in various ways besides determining optimal investment strategies, amongst the other to run sensitivity studies to assess optimal DSR price level, to quantify financial and technical risks associated to specific interventions and suitability of an asset portfolio to meet relevant requirements, to value the impact of inclusion of external costs (for instance, from losses, emissions, reliability metrics, etc.) into CBA, and so forth.

5.3. Potential future work

Following relevant feedback from ENWL and upon agreement with them, we envisage to further improve this work in order to publish it as an academic publication. We also believe that our teams at the University would be in an ideal position for the practical implementation of the tool we proposed here, if ENWL decide to follow our suggestion.

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References

[Bachelier 1900]. L. Bachelier, *Theorie de la Speculation*, Paris, 1900.

[Black and Scholes, 1973]. F. Black and M. Scholes, “The Pricing of Options and Corporate Liabilities”, *J. Polit. Econ*. 81, no. 3, 637–654, 1973

[Black, 1976]. F. Black, “The pricing of Commodity Contracts”, *Journal of Financial Economics* 3, 167-179, 1976.

[Carpaneto et al, 2011a] E. Carpaneto, G. Chicco, P. Mancarella, and A. Russo, “Cogeneration planning under uncertainty. Part I: Multiple time frame approach”, *Applied Energy* 88, Issue 4, April 2011, 1059-1067.

[Carpaneto et al, 2011b] E. Carpaneto, G. Chicco, P. Mancarella, and A. Russo, “Cogeneration planning under uncertainty. Part II: Decision theory-based assessment of planning alternatives”, *Applied Energy* 88, Issue 4, April 2011, 1075-1083.

[Cesena and Mancarella, 2013]. E.A. Martinez-Cesena and P. Mancarella, *Capacity to Customers (C2C) Deliverable 2.1: Methodology for the Cost Benefit Analysis of the C2C Solution*, December 2013.

[Cesena, 2012]. E. A. M. Cesena, “Real Options Theory Applied to Renewable Energy Generation Projects Planning”, *PhD Dissertation*, The University of Manchester, 2012.

[Data and Mathews, 2007]. S. Mathews, V. Datar, and B. Johnson, “A Practical Method for Valuing Real Options: The Boeing Approach,” *J. Appl. Corp. Financ.* 19, no. 2, 94–105, 2007.

[Hoff et al, 1996]. T. E. Hoff, H. J. Wenger and B. K. Farmer, “Distributed generation. An alternative to electric utility investments capacity”, *Energy Policy* 24. No. 2, 137-147, 1996.

[JRC, 2011]. Joint Regulators Group (JRG), *Discounting for CBAs involving private investment, but public benefit*, October 2011.

[Mun, 2006]. J. Mun, *Real Options Analysis: Tools and Techniques for Valuing Strategic Investments and Decisions*, John Wiley & Sons, Hoboken, New Jersey, 2006

[NGET, 2013]. National Grid Electricity Transmission (NGET), *Network Development Policy*, April 2013.

[Ofgem, 2013]. Ofgem, *Strategy decision for the RIIO-ED1 electricity distribution price control - Reliability and safety*, March 2013.

[Wang et al, 2004]. T. Wang, R. De Neufville, and E. S. Division, “Building Real Options into Physical Systems with Stochastic Mixed-Integer Programming”, in *8th Real Options Annual International Conference*, 2004, pp. 1–35.